Further reading

Jackson, J.D., 1975. *Classical Electrodynamics*, Second Edition. Wiley, New York.
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Appendix C. Systems of Units in Nonlinear Optics

There are several different systems of units that are commonly used in nonlinear optics. In this appendix we describe these different systems and show how to convert among them. For simplicity we restrict the discussion to a medium with instantaneous response so that the nonlinear susceptibilities can be taken to be dispersionless. Clearly the rules derived here for conversion among the systems of units are the same for a dispersive medium.

In the gaussian system of units, the polarization $\tilde{P}(t)$ is related to the field strength $\tilde{E}(t)$ by the equation

$$\tilde{P}(t) = \chi^{(1)}\tilde{E}(t) + \chi^{(2)}\tilde{E}^2(t) + \chi^{(3)}\tilde{E}^3(t) + \cdots. \tag{C.1}$$

In the gaussian system, all of the fields $\tilde{E}$, $\tilde{P}$, $\tilde{D}$, $\tilde{B}$, $\tilde{H}$, and $\tilde{M}$ have the same units; in particular, the units of $\tilde{P}$ and $\tilde{E}$ are given by

$$[\tilde{P}] = [\tilde{E}] = \frac{\text{statvolt}}{\text{cm}} = \frac{\text{statcoulomb}}{\text{cm}^2} = \left(\frac{\text{erg}}{\text{cm}^3}\right)^{1/2}. \tag{C.2}$$

Consequently, we see from Eq. (C.1) that the dimensions of the susceptibilities are as follows:

$$\chi^{(1)} \text{ is dimensionless,} \tag{C.3a}$$

$$[\chi^{(2)}] = \left[\frac{1}{\tilde{E}}\right] = \frac{\text{cm}}{\text{statvolt}} = \left(\frac{\text{erg}}{\text{cm}^3}\right)^{-1/2}, \tag{C.3b}$$

$$[\chi^{(3)}] = \left[\frac{1}{\tilde{E}^2}\right] = \frac{\text{cm}^2}{\text{statvolt}^2} = \left(\frac{\text{erg}}{\text{cm}^3}\right)^{-1}. \tag{C.3c}$$

The units of the nonlinear susceptibilities are often not stated explicitly in the gaussian system of units; one rather simply states that the value is given in electrostatic units (esu).

While there are various conventions in use regarding the units of the susceptibilities in the SI system, by far the most common convention is to replace

Eq. (C.1) by

$$\tilde{P}(t) = \epsilon_0 \left[\chi^{(1)}\tilde{E}(t) + \chi^{(2)}\tilde{E}^2(t) + \chi^{(3)}\tilde{E}^3(t) + \cdots\right], \tag{C.4}$$

where

$$\epsilon_0 = 8.85 \times 10^{-12}\ \text{F/m} \tag{C.5}$$

denotes the permittivity of free space. Since the units of $\tilde{P}$ and $\tilde{E}$ in the MKS system are

$$[\tilde{P}] = \frac{\text{C}}{\text{m}^2}, \tag{C.6a}$$

$$[\tilde{E}] = \frac{\text{V}}{\text{m}}, \tag{C.6b}$$

and since 1 farad is equal to 1 coulomb per volt, it follows that the units of the susceptibilities are as follows:

$$\chi^{(1)} \text{ is dimensionless}, \tag{C.7a}$$

$$[\chi^{(2)}] = \left[\frac{1}{\tilde{E}}\right] = \frac{\text{m}}{\text{V}}, \tag{C.7b}$$

$$[\chi^{(3)}] = \left[\frac{1}{\tilde{E}^2}\right] = \frac{\text{m}^2}{\text{V}^2}. \tag{C.7c}$$

C.1. Conversion between the Systems

In order to facilitate conversion between the two systems just introduced, we express the two defining relations (C.1) and (C.4) in the following forms:

$$\tilde{P}(t) = \chi^{(1)}\tilde{E}(t)\left[1 + \frac{\chi^{(2)}\tilde{E}(t)}{\chi^{(1)}} + \frac{\chi^{(3)}\tilde{E}^2(t)}{\chi^{(1)}} + \cdots\right] \text{(gaussian)}, \tag{C.1$'$}$$

$$\tilde{P}(t) = \epsilon_0\chi^{(1)}\tilde{E}(t)\left[1 + \frac{\chi^{(2)}\tilde{E}(t)}{\chi^{(1)}} + \frac{\chi^{(3)}\tilde{E}^2(t)}{\chi^{(1)}} + \cdots\right] \text{(MKS)}. \tag{C.4$'$}$$

The power series shown in square brackets must be identical in each of these equations. However, the values of $\tilde{E}$, $\chi^{(1)}$, $\chi^{(2)}$, and $\chi^{(3)}$ are different in different systems. In particular, from Eqs. (C.2) and (C.5) and the fact that 1 statvolt = 300 V, we find that

$$\tilde{E}\ \text{(MKS)} = 3 \times 10^4\,\tilde{E}\ \text{(gaussian)}. \tag{C.8}$$

To determine how the linear susceptibilities in the gaussian and MKS systems are related, we make use of the fact that for a linear medium the displacement is given in the gaussian system by

$$\tilde{D} = \tilde{E} + 4\pi \tilde{P} = \tilde{E}\big(1 + 4\pi \chi^{(1)}\big), \tag{C.9a}$$

and in the MKS system by

$$\tilde{D} = \epsilon_0 \tilde{E} + \tilde{P} = \epsilon_0 \tilde{E}\big(1 + \chi^{(1)}\big). \tag{C.9b}$$

We thus find that

$$\chi^{(1)}\ (\text{MKS}) = 4\pi \chi^{(1)}\ (\text{gaussian}). \tag{C.10}$$

Using Eqs. (C.8) and (C.9a)–(C.9b), and requiring that the power series of Eqs. (C.1′) and (C.4′) be identical, we find that the nonlinear susceptibilities in our two systems of unit are related by

$$\begin{aligned} \chi^{(2)}\ (\text{MKS}) &= \frac{4\pi}{3 \times 10^4} \chi^{(2)}\ (\text{gaussian}) \\ &= 4.189 \times 10^{-4} \chi^{(2)}\ (\text{gaussian}), \end{aligned} \tag{C.11}$$

$$\begin{aligned} \chi^{(3)}\ (\text{MKS}) &= \frac{4\pi}{(3 \times 10^4)^2} \chi^{(3)}\ (\text{gaussian}) \\ &= 1.40 \times 10^{-8} \chi^{(3)}\ (\text{gaussian}). \end{aligned} \tag{C.12}$$

Appendix D. Relationship between Intensity and Field Strength

In the gaussian system of units, the intensity associated with the field

$$\tilde{E}(t) = E e^{-i\omega t} + \text{c.c.} \tag{D.1}$$

is

$$I = \frac{nc}{2\pi} |E|^2, \tag{D.2}$$

where n is the refractive index, $c = 3 \times 10^{10}$ cm/sec is the speed of light in vacuum, I is measured in erg/cm^2 sec, and E is measured in statvolts/cm.

In the MKS system, the intensity of the field described by Eq. (D.1) is given by

$$I = 2n \left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} |E|^2 = \frac{2n}{Z_0} |E|^2 = 2n\epsilon_0 c |E|^2, \tag{D.3}$$

where $\epsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, and $Z_0 = 377\ \Omega$. I is measured in W/m^2, and E is measured in V/m. Using these relations we can

TABLE D.1 Relation between field strength and intensity

Conventional		Gaussian (cgs)		SI (mks)	
I		I	E	I	E
		(erg/cm^2 sec)	(statvolt/cm)	(W/m^2)	(V/m)
1 kW/m^2		10^6	0.0145	10^3	4.34×10^2
	1 W/cm^2	10^7	0.0458	10^4	1.37×10^3
1 MW/m^2		10^9	0.458	10^6	1.37×10^4
	1 kW/cm^2	10^{10}	1.45	10^7	4.34×10^4
1 GW/m^2		10^{12}	1.45×10	10^9	4.34×10^5
	1 MW/cm^2	10^{13}	45.8	10^{10}	1.37×10^6
1 TW/m^2		10^{15}	4.58×10^2	10^{12}	1.37×10^7
	1 GM/cm^2	10^{16}	1.45×10^3	10^{13}	4.34×10^7
1 ZW/m^2		10^{18}	1.45×10^4	10^{15}	4.34×10^8
	1 TW/cm^2	10^{19}	4.85×10^4	10^{16}	1.37×10^9

obtain the results shown in Table D.1. As a numerical example, a pulsed laser of modest energy might produce a pulse energy or $Q = 1$ mJ with a pulse duration of $T = 10$ nsec. The peak laser power would then be of the order of $P = Q/T = 100$ kW. If this beam is focused to a spot size of $w_0 = 100$ µm, the pulse intensity will be $I = P/\pi w_0^2 \simeq 0.3$ GW/cm^2.

Appendix E. Physical Constants

TABLE E.1 Physical constants in the cgs and SI systems

Constant	Symbol	Value	Gaussian (cgs) [a]	SI (mks) [a]
Speed of light in vacuum	c	2.998	10^{10} cm/sec	10^8 m/sec
Elementary charge	e	4.803	10^{-10} esu	
		1.602		10^{-19} C
Avogadro number	N_A	6.023	10^{23} mol	10^{23} mol
Electron rest mass	$m = m_e$	9.109	10^{-28} g	10^{-31} kg
Proton rest mass	m_p	1.673	10^{-24} g	10^{-27} kg
Planck constant	h	6.626	10^{-27} erg sec	10^{-34} J sec
	$\hbar = h/2\pi$	1.054	10^{-27} erg sec	10^{-34} J sec
Fine structure constant [b]	$\alpha = e^2/\hbar c$	1/137	–	–
Compton wavelength of electron	$\lambda_C = h/mc$	2.426	10^{-10} cm	10^{-12} m
Rydberg constant	$R_\infty = me^4/2\hbar^2$	1.09737	10^5 cm^{-1}	10^7 m^{-1}
Bohr radius	$a_0 = \hbar^2/me^2$	5.292	10^{-9} cm	10^{-11} m
Electron radius [b]	$r_e = e^2/mc^2$	2.818	10^{-13} cm	10^{-15} m
Bohr magneton [b]	$\mu_S = eh/2m_ec$	9.273	10^{-21} erg/G	10^{-24} J/T
		$\Rightarrow$	1.4 MHz/G	

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